Variational Method for Planar Transmission Lines with Anisotropic Magnetic Media

TOSHIHIDE KITAZAWA, SENIOR MEMBER, IEEE

Abstract —**Upper and lower bound expressions of line inductances are presented for the first time for planar transmission lines with anisotropic magnetic media. Green's functions are derived for the general structure by using recurrent relations. Numerical examples of striplines and coplanar waveguide are presented, and the calculated values of the quasi-static characteristics of stripline on an anisotropic magnetic substrate with the tensor permeability are in good agreement with measured values.**

I. Introduction

A LARGE NUMBER of papers have been devoted to the analysis of various types of planar transmission lines with dielectric media. Among them, the quasi-static analysis gives reasonable results with less computational effort, and when based on the variational principle, it provides the margin of error by calculating the upper and lower bounds on the exact value, and the accuracy of the numerical computation can be improved systematically [1]–[11]. The stationary expressions have been obtained for various multiconductor transmission lines with isotropic and/or anisotropic dielectric media. Recently, high-frequency magnetics has been the subject of increasing interest for the use of microwave and millimeter-wave integrated circuits [12], [13]. A simple structure of single microstrip on a magnetic substrate was investigated by employing the duality relationship [14]–[16]. This relationship was extended to more general structures with isotropic magnetic media recently [17], but there is still no quasi-static method available for the case with anisotropic magnetic media.

This paper presents a quasi-static formulation procedure which provides the stationary expressions and both upper and lower bound expressions of line inductances for the general structure with anisotropic magnetic media. Some numerical examples of striplines and coplanar waveguide are presented to show the validity of the method.

II. Theory

A. Recurrent Relation for Multilayered Anisotropic Magnetic Media

The general structure of multilayered anisotropic media with line conductors (Fig. 1) is used to describe the quasi-static formulation procedure. The ith layer may be a

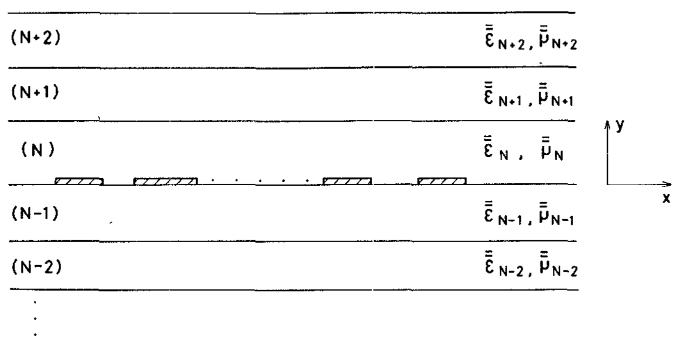


Fig. 1. General structure of multilayered anisotropic media with line conductors.

general anisotropic medium, whose permittivity and permeability are given by the general two-dimensional tensors of the form

$$\bar{\bar{\epsilon}}_i = \begin{bmatrix} \epsilon_{i,xx} & \epsilon_{i,xy} \\ \epsilon_{i,yx} & \epsilon_{i,yy} \end{bmatrix} \epsilon_0 \tag{1a}$$

$$\bar{\bar{\mu}}_i = \begin{bmatrix} \mu_{i,xx} & \mu_{i,xy} \\ \mu_{i,yx} & \mu_{i,yy} \end{bmatrix} \mu_0. \tag{1b}$$

As for nonmagnetic problems, quasi-static characteristics are described only by line capacitances, and the variational method has been successfully applied to obtain the line capacitances of various types of transmission lines with isotropic and/or anisotropic media [1]–[11]. Since line inductances are needed to characterize the transmission lines with magnetic media, the formulation procedure is presented to obtain line inductances with general anisotropic magnetic media in the following.

First, we will derive the recurrent relation, which will serve to construct Green's functions for multilayered anisotropic magnetic media. We define the following quantity at the lower surface of layer i (Fig. 2):

$$\tilde{Y}_i = \frac{|\alpha| \mu_{i,e} \mu_0}{j\alpha} \cdot \frac{\tilde{H}_x}{\tilde{B}_y}\bigg|_{y = y_i + 0} \tag{2}$$

where

$$\mu_{i,e} = \sqrt{\mu_{i,xx}\mu_{i,yy} - \left(\frac{\mu_{i,xy} + \mu_{i,yx}}{2}\right)^2} \tag{3}$$

and $\tilde{H}_x$ and $\tilde{B}_y$ are the Fourier transforms of the magnetic field H_x and the magnetic flux density B_y in layer i

Manuscript received February 1, 1989; revised June 12, 1989.
The author is with the Department of Electronic Engineering, Kitami Institute of Technology, Kitami 090, Japan.
IEEE Log Number 8930523.

0018-9480/89/1100-1749$01.00

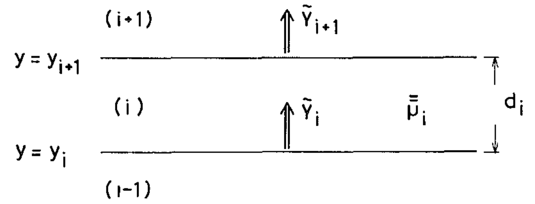


Fig. 2. The ith layer of stratified anisotropic media.

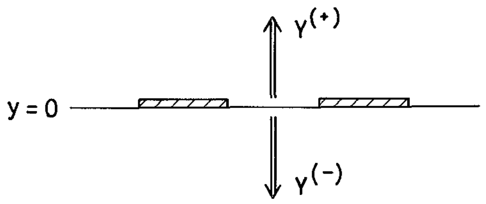


Fig. 3. $Y^{(+)}$ and $Y^{(-)}$.

$(y_{i+1} > y > y_i)$, respectively:

$$\tilde{H}_x(\alpha; y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} H_x(x, y) e^{j\alpha x} \, dx \tag{4}$$

$$\tilde{B}_y(\alpha; y) = \mu_{i,xy}\mu_0 \tilde{H}_x + \mu_{i,yy}\mu_0 \tilde{H}_y. \tag{5}$$

Considering the continuity conditions at the $y = y_{i+1}$ plane (Fig. 2), we obtain the following recurrent relation with respect to $\tilde{Y}_i$:

$$\tilde{Y}_i = \frac{\frac{\mu_{i,e}}{\mu_{i+1,e}} \tilde{Y}_{i+1}\{1 - q_i \tanh(p_i d_i)\} + \tanh(p_i d_i)}{\frac{\mu_{i,e}}{\mu_{i+1,e}} \tilde{Y}_{i+1}(1 - q_i^2) \tanh(p_i d_i) + q_i \tanh(p_i d_i) + 1} \tag{6}$$

where

$$q_i = j \frac{\mu_{i,xy} + \mu_{i,yx}}{2\mu_{i,yy}} \alpha \tag{7}$$

$$p_i = \frac{\mu_{i,e}}{\mu_{i,yy}} |\alpha|. \tag{8}$$

The quantity $\tilde{Y}_i$ at the conductor plane $(y = +0)$, $Y^{(+)}$ (Fig. 3), can be obtained by using the above recurrent relation. A similar recurrent relation holds in the lower region $y < 0$, and the quantity $\tilde{Y}_i$ at the $y = -0$ plane, $Y^{(-)}$, can be determined. Considering the boundary condition at the conductor plane $y = 0$

$$\tilde{H}_x(\alpha; y = +0) - \tilde{H}_x(\alpha; y = -0) = -\tilde{i}_z \tag{9a}$$

$$\tilde{B}_y(\alpha; y = +0) = \tilde{B}_y(\alpha; y = -0) \tag{9b}$$

we obtain

$$\frac{j\alpha}{|\alpha|\mu_0} \left\{ \frac{1}{\mu_{N,e}} Y^{(+)} + \frac{1}{\mu_{N-1,e}} Y^{(-)} \right\} \tilde{B}_y(\alpha; y = 0) = -\tilde{i}_z \tag{10}$$

where $\tilde{i}_z$ is the Fourier transform of the current distribution $i_z(x)$ on the conductors:

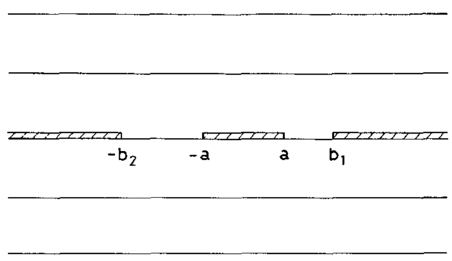


Fig. 4. Asymmetrical coplanar waveguide (A-CPW) with layered anisotropic media.

$$\tilde{i}_z(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} i_z(x) e^{j\alpha x} \, dx. \tag{11}$$

Thus, the current distribution $i_z(x)$ on the conductors can be expressed in terms of the magnetic flux density at the conductor plane $b_y(x)$:

$$i_z(x) = \iint_{-\infty}^{\infty} f(\alpha) e^{-j\alpha(x - x')} b_y(x') \, d\alpha \, dx' \tag{12}$$

with

$$f(\alpha) = \frac{-1}{2\pi} \{Y_U(\alpha) + Y_L(\alpha)\} \tag{13}$$

$$Y_U(\alpha) = \frac{j}{|\alpha|\mu_0\mu_{N,e}} Y^{(+)} \tag{14}$$

$$Y_L(\alpha) = \frac{j}{|\alpha|\mu_0\mu_{N-1,e}} Y^{(-)} \tag{15}$$

or the magnetic flux density can be expressed in terms of the current distribution as follows:

$$b_y(x) = \iint_{-\infty}^{\infty} g(\alpha) e^{-j\alpha(x - x')} i_z(x') \, d\alpha \, dx' \tag{16}$$

with

$$g(\alpha) = \frac{-1}{2\pi} \cdot \frac{1}{Y_U(\alpha) + Y_L(\alpha)}. \tag{17}$$

B. Upper and Lower Bound Expressions for the Line Inductances

The formulation procedures to obtain the stationary expressions are essentially common to various planar transmission lines, and the procedure is explained only for asymmetrical coplanar waveguide (A-CPW: Fig. 4) as an example. Numerical results of single and coupled striplines as well as CPW will be presented in a later section.

1) Upper Bound Expression for the Line Inductances: An upper bound expression for the line inductances can be derived by using (16). First, we introduce the total magnetic flux through the semi-infinite region $x > x_1$:

$$\Phi(x_1) = \int_{x_1}^{\infty} b_y(x) \, dx. \tag{18}$$

$\Phi(x_1)$ should be constant when x_1 lies on the conductors, i.e.,

$$\Phi(x_1)=0 \qquad (x_1<-b_2, b_1<x_1)$$
$$\Phi(x_1)=\Phi_0 \qquad (|x_1|<a) \tag{19}$$

where Φ_0 is the total magnetic flux through the slot:

$$\Phi_0=-\int_{-b_2}^{-a} b_y(x)\,dx=\int_a^{b_1} b_y(x)\,dx. \tag{20}$$

Multiplying (19) by $i_z(x_t)$ and integrating over the conductors, we get

$$0=\int_{b_1}^{\infty}\Phi(x_1)i_z(x_1)\,dx_1 \tag{21a}$$

$$I_0\Phi_0=\int_{-a}^{a}\Phi(x_1)i_z(x_1)\,dx_1 \tag{21b}$$

$$0=\int_{-\infty}^{-b_2}\Phi(x_1)i_z(x_1)\,dx_1 \tag{21c}$$

where I_0 is the total current on the center strip and is given by

$$I_0=\int_{-a}^{a} i_z(x)\,dx. \tag{22}$$

Summing (21a) through (21c) and substituting (16) into the resultant equation, we obtain

$$I_0\Phi_0=\int_{-\infty}^{\infty}\Phi(x_1)i_z(x_1)\,dx_1$$
$$=\int\int\int_{-\infty}^{\infty} i_z(x)G_U(\alpha;x|x')i_z(x')\,d\alpha\,dx'\,dx \tag{23}$$

where $G_U(\alpha;x|x')$ is given by

$$G_U(\alpha;x|x')=\frac{g(\alpha)}{j\alpha}e^{-j\alpha(x-x')}. \tag{24}$$

Therefore, the line inductance can be obtained by dividing (23) by I_0^2:

$$L=\frac{\Phi_0}{I_0}$$
$$=\frac{\int\int\int_{-\infty}^{\infty} i_z(x)G_U(\alpha;x|x')i_z(x')\,d\alpha\,dx'\,dx}{\left\{\int_{-a}^{a} i_z(x)\,dx\right\}^2}. \tag{25}$$

It can easily be shown that (25) is stationary and provides the upper bound on the line inductance.

2) Lower Bound Expression for the Line Inductances: For the derivation of the lower bound expression of the line inductances, we will work with the total current on the conductors between x_2 and x_1:

$$I(x_1,x_2)=\int_{x_2}^{x_1} i_z(x)\,dx \tag{26}$$

instead of the total magnetic flux $\Phi(x)$. When both x_1 and x_2 lie in the slot region, $I(x_1,x_2)$ should be constant; e.g., when x_2 lies within the left slot $(-b_2<x_2<-a)$ and x_1 lies within the right slot $(a<x_1<b_1)$, $I(x_1,x_2)$ equals I_0, the total current on the center strip:

$$I_0=I(x_1,x_2)=\int_{x_2}^{x_1} i_z(x)\,dx$$
$$\cdot(-b_2<x_2<-a;\ a<x_1<b_1). \tag{27}$$

We multiply (27) by $b_y(x_1)$ and $b_y(x_2)$ and integrate with respect to x_1 over the right slot $(a<x_1<b_1)$ and x_2 over the left slot $(-b_2<x_2<-a)$, obtaining

$$-I_0\Phi_0^2=\int_a^{b_1} b_y(x_1)\int_{-b_2}^{-a} b_y(x_2)\left\{\int_{x_2}^{x_1} i_z(x)\,dx\right\}dx_2\,dx_1 \tag{28}$$

where Φ_0 is the total magnetic flux through the slot (eq. (20)). Substituting (12) into the resultant equation, we obtain

$$\frac{1}{L}=\frac{I_0}{\Phi_0}$$
$$=\frac{\int\int\int_{-\infty}^{\infty} b_y(x)G_L(\alpha;x|x')b_y(x')\,d\alpha\,dx'\,dx}{\left\{\int_a^{b_1} B_y(x)\,dx\right\}^2} \tag{29}$$

where $G_L(\alpha;x|x')$ is given by

$$G_L(\alpha;x|x')=\frac{f(\alpha)}{j\alpha}e^{-j\alpha(x-x')}. \tag{30}$$

We mention that (29) is stationary and provides the lower bound on the line inductance.

III. Numerical Examples

The line inductance can be evaluated by applying the Ritz procedure to the stationary expressions (25) or (29). In this procedure, the unknown quantities, i.e., $i_z(x)$ and $b_y(x)$, are expanded in terms of the known basis functions. The accurate numerical computation is available with a few basis functions when the singular behavior of the unknown quantities near the conductor edge is approximated properly [3]–[5], [7]–[12]. In the following computations, we have utilized the basis functions which take the edge effect into consideration. A simple structure of symmetrical CPW is considered in Table I to investigate the validity of the present method. A problem of practical interest is the case with a dielectric–magnetic substrate whose ϵ_r or μ_r is not equal to unity, e.g., a demagnetized ferrite substrate. Upper and lower bounds of the characteristic impedance are calculated. In such a case, line capacitances as well as line inductances are required to obtain line parameters. The line capacitances are determined by using the variational method in [3] and [4], and line inductances are obtained by using (25) and (29). Table I includes the case without substrates to compare the numerical results with the exact analytical values [4]. The

TABLE I
CHARACTERISTIC IMPEDANCE (Z_0) OF CPW

ε_r	μ_r	S/W	h/W	L.B.	U.B. (%)	Conformal mapping
12	0.8	0.2	1	88.053	88.093 (0.05)	-
12	0.8	0.5	1	71.238	71.254 (0.02)	-
12	0.8	1.0	1	60.749	60.787 (0.06)	-
1	1	0.2	1	226.925	226.981 (0.02)	226.927
1	1	0.5	1	179.133	179.218 (0.05)	179.134
1	1	1.0	1	147.347	147.536 (0.13)	147.347

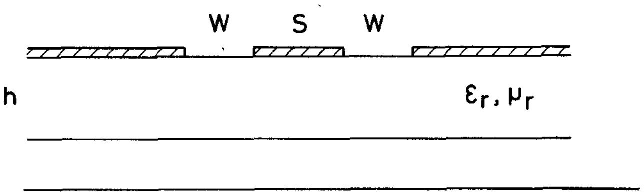
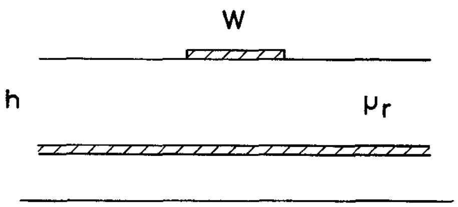


TABLE II
EFFECTIVE RELATIVE PERMEABILITIES (μ_{eff}) OF SINGLE MICROSTRIP

μ_r	W/h	Present (%)	Ref.[14]
0.4	0.2	0.5344 (-0.224)	0.5356
0.4	1.0	0.5088 (-0.157)	0.5096
0.4	4.0	0.4673 (1.411)	0.4608
0.6	0.2	0.7192 (-0.111)	0.7200
0.6	1.0	0.6978 (-0.014)	0.6979
0.6	4.0	0.6623 (0.976)	0.6559
0.8	0.2	0.8715 (-0.023)	0.8717
0.8	1.0	0.8594 (-0.047)	0.8590
0.8	4.0	0.8387 (0.527)	0.8343



difference between the lower and the upper bounds on Z_0 is less than 0.2 percent for all cases calculated.

The effective relative permeabilities μ_{eff} of single microstrip on an isotropic magnetic substrate are presented in Table II. The calculated values of μ_{eff} do not give upper or lower bounds, and only the values obtained by (25) are presented and compared with those obtained by [14, eq. (14)]. An excellent agreement is observed over a wide range of parameters.

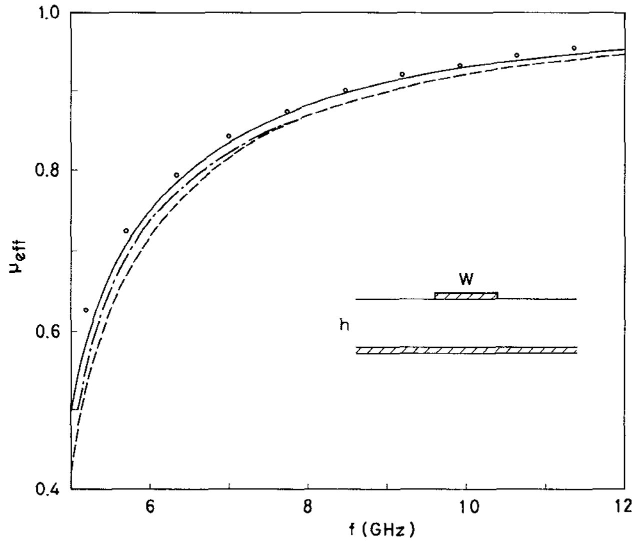


Fig. 5. Effective relative permeability of single microstrip on a partially magnetized garnet substrate: $h = 0.25$ in, $W/h = 0.5$, $4\pi M_s = 1780$ G, $4\pi M_r = 1030$ G; ——— present theory based on the tensor permeability (eq. (31)),— — — — — present theory based on the equivalent scalar permeability (eq. (35)), —·—·— reference [15], ∘ ∘ ∘ measured values [15].

The method is also applied to cases with gyromagnetic media. The relative permeability of a gyromagnetic substrate biased along the direction of propagation (z axis) is given by the following tensor:

$$\bar{\bar{\mu}} = \begin{bmatrix} \mu & -j\kappa \\ j\kappa & \mu \end{bmatrix}. \tag{31}$$

Fig. 5 shows the effective relative permeability of single microstrip on a partially magnetized garnet substrate. The diagonal and off-diagonal elements of the tensor permeability can be expressed as [15], [16]

$$\mu = \mu_{dem} + (1 - \mu_{dem})\left(\frac{4\pi M}{4\pi M_s}\right)^{3/2} \tag{32}$$

$$\kappa = \frac{\gamma(4\pi M)}{\omega} \tag{33}$$

$$\mu_{dem} = \frac{1}{3} + \frac{2}{3}\left\{1 - \gamma\left(\frac{4\pi M_s}{\omega}\right)^2\right\}^{1/2} \tag{34}$$

where γ is the gyromagnetic ratio and $4\pi M_s$ is the saturation magnetization. Numerical values from the present method are compared with the measured values [15]. The present method is based on the quasi-static approximation, and the frequency dependence is incorporated into the tensor elements μ and κ only, but good agreement between calculated and measured values is observed throughout the frequency region considered. Fig. 5 includes the calculated values by [15] (—·—·—) for comparison. They were obtained by using the equivalent scalar permeability, which was presented in [15] with "no physical basis" and is

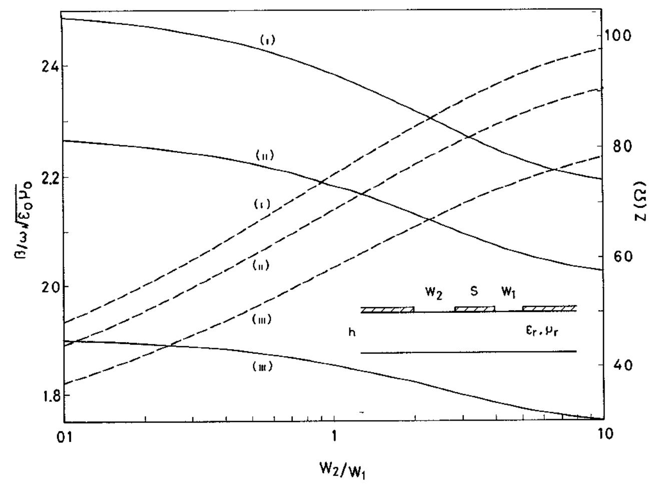
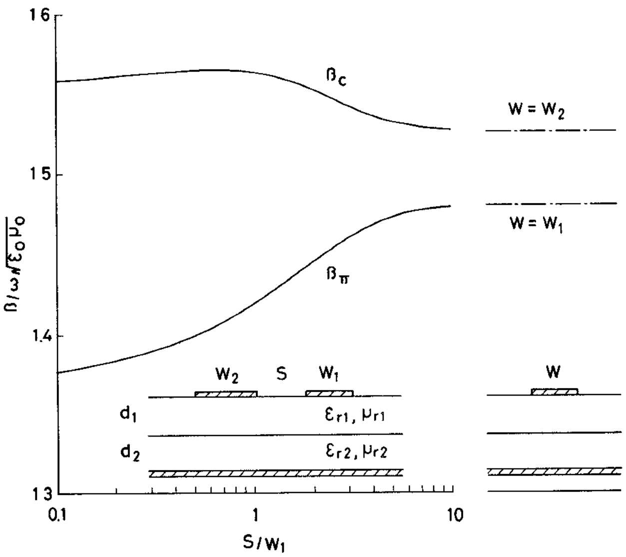
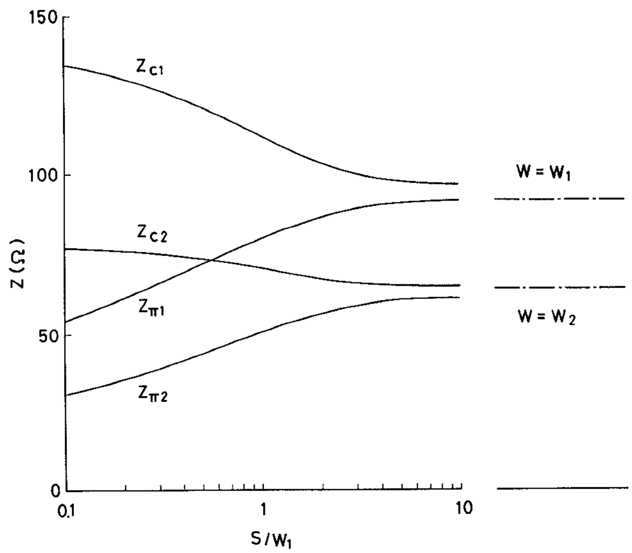


Fig. 6. Line characteristics of A-CPW on a dielectric–magnetic substrate: $\epsilon_r = 12$, $S/W_1 = 0.5$, $h/W_1 = 1$, (i) $\mu_r = 1$, (ii) $\mu_r = 0.7$, (iii) $\mu_r = 0.4$; ——— phase constants, ----- characteristic impedances.



(a)



(b)

Fig. 7. Asymmetric coupled strips with double-layered substrate. (a) Phase constants. (b) Characteristic impedances. $\epsilon_{r1} = 2.6$, $\mu_{r1} = 1.0$, $d_1/W_1 = 0.8$, $\epsilon_{r2} = 12.5$, $\mu_{r2} = 0.4$, $d_2/W_1 = 1.0$, $W_2/W_1 = 2$.

expressed as

$$\mu_{eq} = \mu_L \frac{1}{1 - \frac{1}{7}\sqrt{\frac{h}{W}}\left(\frac{\kappa}{\mu}\right)^2 \ln\left(1 + \frac{1}{\mu_L}\right)} \tag{35}$$

$$\mu_L = \frac{\mu^2 - \kappa^2}{\mu}. \tag{36}$$

Also, the equivalent scalar permeability (35) was introduced into the present method (----) for comparison. Curves based on the equivalent scalar permeability give reasonable results for single stripline. However, we mention that the expression of the equivalent permeability μ_{eq} (eq. (35)) contains the ratio W/h and is applicable only for the single microstrip case.

The present method is quite versatile and is applicable to various types of planar transmission lines. Figs. 6 and 7 show the results obtained for asymmetrical structures. These structures are promising because of the additional flexibilities offered by the asymmetrical configuration and the impedance transform nature. Fig. 6 shows the line characteristics of A-CPW on a dielectric–magnetic substrate. Fig. 7 shows the phase constants and characteristic impedances of asymmetric coupled strips with double-layered substrate as a function of the spacing of strips S/W. The phase constants of the π and c modes converge to the values of the single stripline of widths W_1 and W_2 with the same substrates, respectively (Fig. 7(a)). Then the characteristic impedance of strip for the π mode, $Z_{\pi 1}$, and that of strip for the c mode, Z_{c2}. converge to those of the single strip of widths W_1 and W_2, respectively (Fig. 7(b)).

IV. Conclusions

This paper explains the variational method, which is applicable to various types of planar transmission lines with anisotropic magnetic media. Upper and lower bound expressions of line inductances are presented for planar transmission lines with stratified magnetic media for the first time. Green's functions are derived for the general structure by using a simple recurrent relation. Numerical computations are carried out to confirm the validity of the method. The quasi-static characteristics are calculated for the stripline on an anisotropic magnetic substrate with the tensor permeability, and good agreement is observed between calculated and measured values. Also, the quasi-static characteristics of striplines and coplanar waveguide with dielectric–magnetic substrates are presented.

References

[1] E. Yamashita and R. Mittra, "Variational method for the analysis of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 251–256, Apr. 1968.
[2] K. Araki and Y. Naito, "Upper bound calculations on capacitance of microstrip line using variational method and spectral domain approach," *IEEE Trans. Microwave Theory Tech.*, vol,. MTT-26, pp. 506–509, July 1978.
[3] T. Kitazawa and Y. Hayashi, "Coupled slots on an anisotropic sapphire substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 1035–1040, Oct. 1981.

[4] T. Kitazawa and Y. Hayashi, "Quasi-static characteristics of coplanar waveguide on a sapphire substrate with its optical axis inclined," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 920–922, June 1982.

[5] T. Kitazawa and Y. Hayashi, "Quasistatic characteristics of coplanar waveguide with thick metal coating," *Proc. Inst. Elec. Eng.*, pt.H., vol. 133, pp. 18–20, Feb. 1986.

[6] M. Horno, "Upper and lower bounds on capacitances of coupled microstrip lines with anisotropic substrates," *Proc. Inst. Elec. Eng.*, pt.H., vol. 129, pp. 89–93, June 1982.

[7] T. Kitazawa and R. Mittra, "Analysis of asymmetric coupled striplines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 643–646, July 1985.

[8] Y. Hayashi, T. Kitazawa, and S. Sasaki, "Analysis of coplanar strip lines on an anisotropic substrate using Galerkin's method," *Trans. IECE Japan*, vol. 64-B, no. 7, pp. 666–673, July 1981.

[9] T. Kitazawa, Y. Hayashi, K. Fujita, and H. Mukaihara, "Analysis of broadside-coupled strip lines with anisotropic substrate," *Trans. IECE Japan*, vol. 66-B, no. 9, pp. 1139–1146, Sept. 1983.

[10] T. Kitazawa and Y. Hayashi, "Analysis of unsymmetrical broadside-coupled striplines with anisotropic substrates," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 188–191, Jan. 1986.

[11] T. Kitazawa and R. Mittra, "Quasistatic characteristics of asymmetrical and coupled coplanar-type transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 771–778, Sept. 1985.

[12] M. Geshiro and T. Itoh, "Analysis of double-layered finlines containing a magnetized ferrite," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 1377–1381, Dec. 1987.

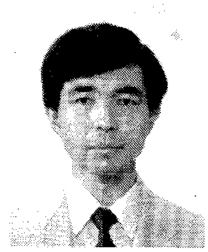
[13] "Special section on microwave magnetics," *Proc. IEEE*, vol. 76, pp. 121–200, Feb. 1988.

[14] R. A. Pucel and D. J. Masse, "Microstrip propagation on magnetic substrates-Part I: Design theory," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 304–308, May 1972.

[15] R. A. Pucel and D. J. Masse, "Microstrip propagation on magnetic substrates—Part II: Experiment," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 309–313, May 1972.

[16] M. S. Sodha and N. C. Srivastava, *Microwave Propagation in Ferrimagnetics*. New York: Plenum, 1981, pp. 377–380.

[17] J. R. Mautz, R. F. Harrington, and C. G. Hsu, "The inductance matrix of a multiconductor transmission line in multiple magnetic media," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 1293–1295, Aug. 1988.



Toshihide Kitazawa (M'84–SM'86) was born in Sapporo, Japan, on December 1, 1949. He received the B.E., M.E., and D.E. degrees in electronics engineering from Hokkaido University, Sapporo, Japan, in 1972, 1974, and 1977, respectively.

He was a Post-Doctoral Fellow of the Japan Society for the Promotion of Science from 1979 to 1980. Since 1980, he has been an Associate Professor of Electronic Engineering at the Kitami Institute of Technology, Kitami, Japan. From 1982 to 1984, he was a Visiting Assistant Professor of Electrical Engineering at the University of Illinois, Urbana.

Dr. Kitazawa is a member of the Information Processing Society of Japan and the Institute of Electronics, Information and Communication Engineers of Japan.